CHAPTER 1 -- MATH REVIEW

1.1) Both *Parts a* and *b* are straight vector addition, graphical style. You need a centimeter stick and protractor to do them.

a.) In this case, the velocity component contributed by the rower is a vector directed straight across the river. It has a magnitude of 5 miles per hour. The velocity component contributed by the river's motion is directed downward. It has a magnitude of 2 miles per hour. Vectorial addition of the two vectors yields the net velocity of the boat during the crossing (see Figure 1). A protractor should be used to measure the angle. It turns out to be -22^o (minus because it is *below* the reference line straight across the river). A centimeter stick measures the resultant at 5.4 centimeters. Multiplying by the scaling factor, we get a net velocity vector of 5.4 miles per hour.

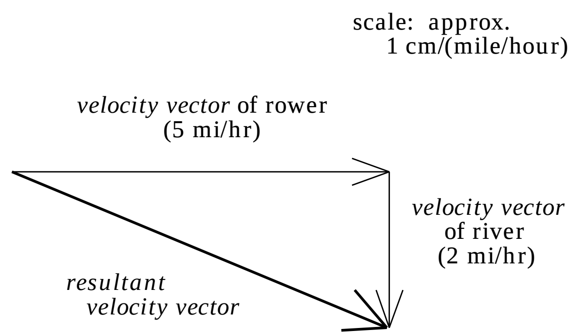

FIGURE 1

Note: The Xerox machine ever-so-slightly miniaturizes whatever it copies. When I used the original to make my measurements, I got the answer quoted above. If you use a centimeter stick and protractor on the Xeroxed copy, you will get an answer that is a bit off. If you do the problem on your own, you should get the same solution as I did to a good approximation.

b.) The direction of any object's *net velocity* is the same as the direction of the object's *net motion*. We want the net motion to be along the direction shown in Figure I on page 22. The question that is really being asked is: "What boat-velocity vector

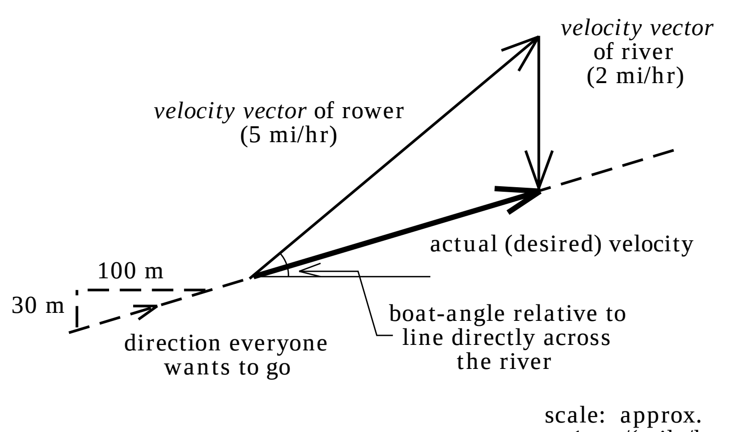

FIGURE 2

must exist if, when added to a water-velocity vector, yields the net-desired-boat-motion?" Clearly it must be upstream. Playing with a compass and protractor, the solution is approximately 40^o (see Figure 2 on the previous page) and the velocity is approximately 4 miles/hour.

1.2) This is also a problem for graphical vector addition.

a.) The vectors involved are shown to the right. A protractor was used to determine an angle of approximately 17^o north of west.

b.) The diagram is to scale. The resultant, measured by a centimeter stick, is approximately 5.6 centimeters. Multiplying by the scaling factor yields (5.6)(20) = 112 miles. This is the net displacement.

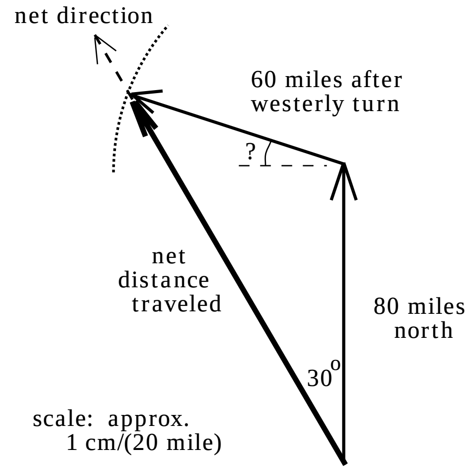

FIGURE 3

Note: Once again, if you use a centimeter stick and the sketch in Figure 3 to follow my steps in doing this problem, you will probably find your result and my result differing just a bit. I did the problem off the original drawing. Xeroxing slightly shrinks the material being copied.

1.3) The graph of vectors $\boldsymbol{P}$ and $\boldsymbol{T}$ are shown in Figure 4.

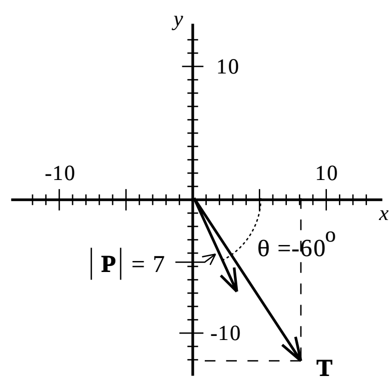

FIGURE 4

1.4) Using the graph given in problem-Figure II:

in unit vector notation: $\mathbf{A} = 2.25\mathbf{i} + .5\mathbf{j}$.
$\mathbf{C} = -.72\mathbf{i} - \mathbf{j}$.

in polar notation: $\mathbf{B} = 1.3 \angle 125^o$.
$\mathbf{D} = 2.6 \angle -20^o$.

Note: I've made the problem tricky as far as the two polar angles go. <u>All angles</u> must be measured either clockwise or counterclockwise FROM THE +*x AXIS*!

1.5) Each axis has its own scale. If the scales had been the same, relative lengths and angles would have made sense. They aren't, hence they don't.

1.6) This is all straight vector manipulation. You should be able to do this kind of thing in your sleep!

a.) $-(1/3)\mathbf{A} = (-1/3)((-8\mathbf{i} + 12\mathbf{j})$
$= (8/3)\mathbf{i} - 4\mathbf{j}$.

b.) $-6\mathbf{E} = (-6)((12 - 225^o)$
$= (72 \angle 45^o)$.

Note: in polar notation, a *scalar* multiplies the *magnitude* of the vector while a *negative sign* adds (or subtracts) 180^o to the *angle* of the vector.

c.) $\mathbf{A} + \mathbf{B} - \mathbf{C} = (-8\mathbf{i} + 12\mathbf{j}) + (-4\mathbf{i} - 3\mathbf{j}) - (5\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$
$= -17\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$.

d.) A bit of trig and Figure 5 show ***E's*** unit vector equivalent. Written in *unit vector notation*:

$\mathbf{E} = -8.48\mathbf{i} - 8.48\mathbf{j}$.

Note: The negative signs had to be put in manually.

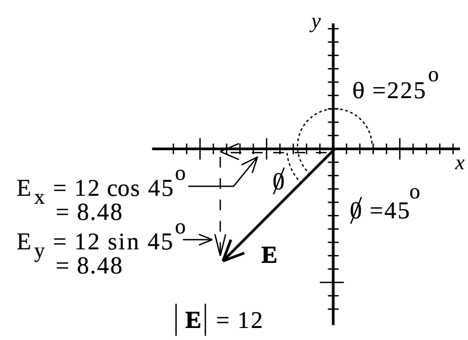

FIGURE 5

e.) A bit of trig and Figure 6 shows **F**'s *unit vector* equivalent to be:

$\mathbf{F} = -.518\mathbf{i} + 1.93\mathbf{j}$.

Note: Again, the negative signs had to be placed manually.

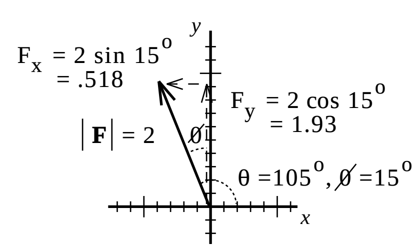

FIGURE 6

f.) The Pythagorean relationship, a bit of trig, and Figure 7 shows **A**'s *polar* equivalent to be:

$$\mathbf{A} = 14.4 \angle 123.7^{o}.$$

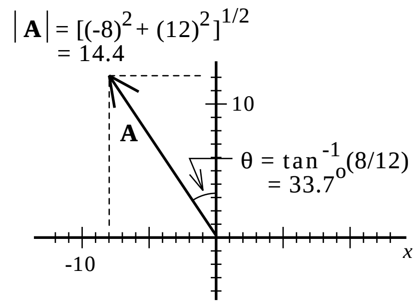

FIGURE 7

Note: *Tan*$^{-1}$ *(8/12) = 33.7*^{o}. Adding that to 90^{o} gives us the calculated 123.7^{o}. An alternate way to do the problem: take the actual components, minus signs and all, and determine *tan*$^{-1}$ (A_y/A_x). That is, *tan*$^{-1}$ *(12/-8) = -56.3*^{o}. This is a *fourth quadrant angle,* whereas our vector is a *second quadrant angle.* To adjust the situation, we must add 180^{o} to get the appropriate 123.7^{o} angle. Either way will do.

g.) The Pythagorean relationship, a bit of trig, and Figure 8 shows **B**'s *polar* equivalent to be:

$$\mathbf{B} = 5 \angle 216.9^{o}.$$

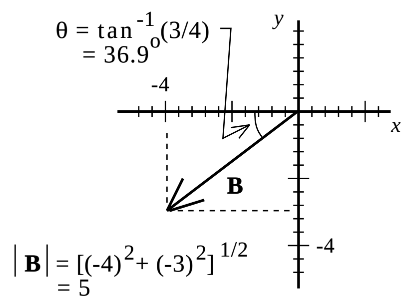

FIGURE 8

Note: *Tan*$^{-1}$ *(3/4) = 36.9*^{o}. Added to 180^{o} gives us the appropriate 216.9^{o}. An alternate way to do the problem: take the actual components, *minus signs and all*, and determine the *tan*$^{-1}$ (A_y/A_x), or *tan*$^{-1}$*(-3/-4) = 36.9*^{o}. This is a *first quadrant angle*, whereas our vector is a *third quadrant angle*. To adjust the situation, we must add 180^{o} to get the appropriate 216.9^{o} angle. Either way will do.

h.) $\mathbf{A \cdot C} = A_xC_x + A_yC_y + A_zC_z$

$$= (-8)(5) + (12)(6) + (0)(-7)$$
$$= 32.$$

i.) $\mathbf{D \cdot E} = |\mathbf{D}||\mathbf{E}|\cos\phi$

$$= (7)\ (12)\cos 75^{o}$$
$$= 21.74.$$

j.)

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 12 & 0 \\ -4 & -3 & 0 \end{vmatrix}$$

$$= 0\mathbf{i} + 0\mathbf{j} + [(-8)(-3) - (12)(-4)]\mathbf{k}$$
$$= 72\mathbf{k}.$$

k.)

$$\mathbf{C} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & -7 \\ -4 & -3 & 0 \end{vmatrix}$$

$$= [(6)(0) - (-7)(-3)]\mathbf{i} + [(-7)(-4) - (5)(0)]\mathbf{j} + [(5)(-3) - (6)(-4)]\mathbf{k}$$
$$= -21\mathbf{i} + 28\mathbf{j} + 9\mathbf{k}.$$

l.) $\mathbf{D}\mathrm{x}\mathbf{E} = |\mathbf{D}||\mathbf{E}|\sin\phi\ (-\mathbf{k})$
$= (7)\ (12)\ \sin 75^{o}$
$= -81.1\mathbf{k}.$

Note: The *-**k** direction* came from using the *right-hand rule* on **D**x**E**.

1.7.) A *dot product* gives you a scalar equal to the *magnitude of **A*** times the magnitude of the component of **B** *parallel to **A***. OR:

It gives you a scalar equal to the *magnitude of **B*** times the magnitude of the component of **A** *parallel to **B***.

1.8.) A *cross product* gives you a vector whose magnitude is equal to the *magnitude of **A*** times the magnitude of the component of **B** *perpendicular to **A***. OR:

It gives you a vector whose magnitude is equal to the *magnitude of **B*** times the magnitude of the component of **A** *perpendicular to **B***.

The direction of the cross product is perpendicular to the plane defined by **A** and **B**.